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Mathematics

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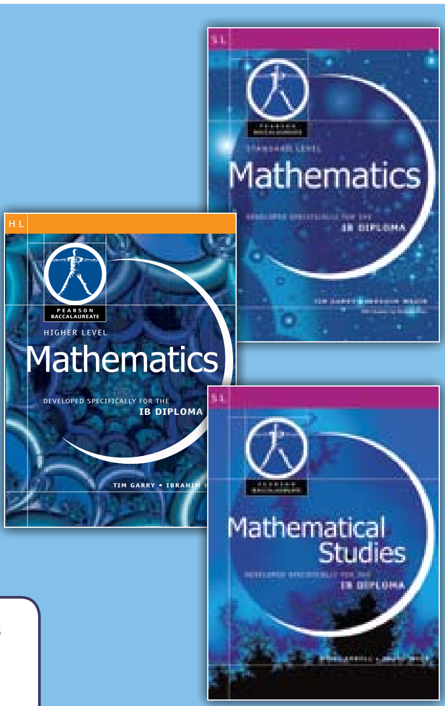

Standard level

Written by Ibrahim Wazir and Tim Garry

Contents:

Introduction
1. Algebra
2. Sequences and Series
3. Functions
4. Exponents and Logarithms
5. Matrices
6. Trigonometry I
7. Vectors I
8. Descriptive Statistics
9. Introductory Probability
10. Differential Calculus I
11. Differential Calculus II - Applications
12. Vectors II
13. Trigonometry II
14. Differential Calculus III
15. Integral Calculus
16. Further Statistics & Probability
17. Portfolios - Ideas & Examples
18. Practice IB Exams

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Higher Level

Written by Ibrahim Wazir and Tim Garry

Contents:

Introduction
1. Fundamentals
2. Functions
3. Algebraic Functions, Equations and Inequalities
4. Sequences and Series
5. Exponential and Logarithmic Functions
6. Matrix Algebra
7. Trigonometric Functions and Equations
8. Triangle Trigonometry
9. Vectors
10. Complex Numbers
11. Statistics
12. Probability
13. Differential Calculus I Fundamentals
14. Vectors, Lines and Planes
15. Differential Calculus II Further Techniques and Applications
16. Integral Calculus
17. Probability Distributions
18. Extended Problems, Investigations and Projects
19. Sample Examination Papers
20. Theory of Knowledge

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Additionally, the options available for book adopters online at no extra cost: Topic 8 - Statistics and probability, Topic 9 - Sets, relations and groups, Topic 10 - Series and differential equations, Topic 11 - Discrete mathematics

Mathematical Studies

Written by Ron Carrell and David Wees

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Introduction
1. Presumed Knowledge
2. Numbers
3. Functions I (linear, quadratic, exponential)
4. Functions II (since, cosine, unfamiliar)
5. Geometry I (basic concepts, 2-D, 3-D)
6. Geometry II (solving triangles)
7. Sequences and Series
8. Financial
9. Sets
10. Logic
11. Probability
12. Statistics I (basic concepts)
13. Statistics II (regression line, chi-square)
14. Calculus
15. The Project
16. Practice IB exams

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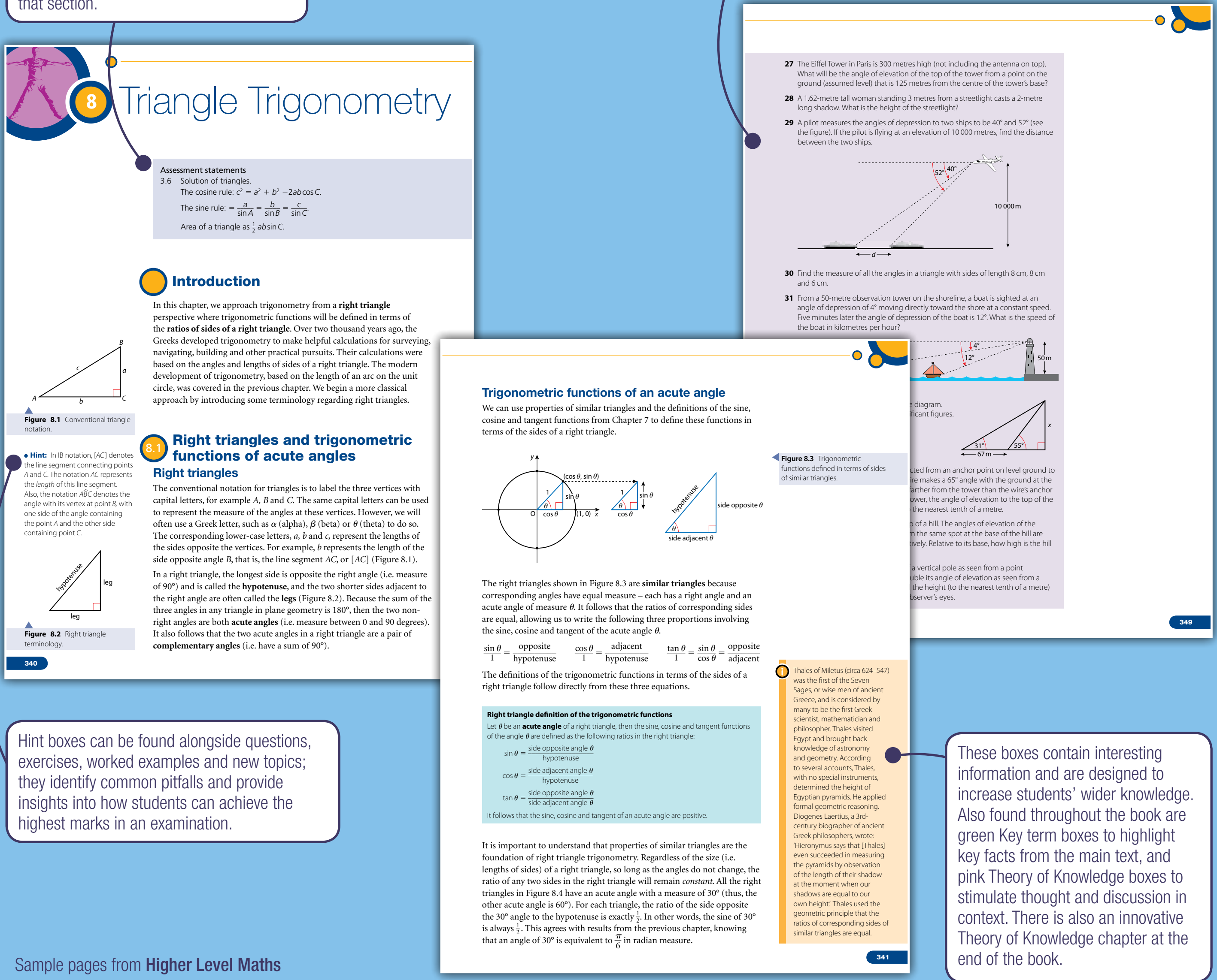
An assessment statement box starts each new section in a chapter. These show the numbered objectives and set out the content and aspects of learning covered in that section.

Exercises are found at the end of each section in a chapter. At the end of each chapter, there is an Exam Practice section containing real IB exam questions.



8 Triangle Trigonometry

Assessment statements
3.6 Solution of triangles.
The cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$.
The sine rule: $= \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
Area of a triangle as $\frac{1}{2}ab\sin C$.

Introduction

In this chapter, we approach trigonometry from a **right triangle** perspective where trigonometric functions will be defined in terms of the **ratios of sides of a right triangle.** Over two thousand years ago, the Greeks developed trigonometry to make helpful calculations for surveying, navigating, building and other practical pursuits. Their calculations were based on the angles and lengths of sides of a right triangle. The modern development of trigonometry, based on the length of an arc on the unit circle, was covered in the previous chapter. We begin a more classical approach by introducing some terminology regarding right triangles.

Figure 8.1 Conventional triangle notation.

- **Hint:** In IB notation, $[AC]$ denotes the line segment connecting points A and C. The notation AC represents the *length* of this line segment. Also, the notation $A\hat{B}C$ denotes the angle with its vertex at point B, with one side of the angle containing the point A and the other side containing point C.

8.1 Right triangles and trigonometric functions of acute angles

Right triangles

The conventional notation for triangles is to label the three vertices with capital letters, for example A, B and C. The same capital letters can be used to represent the measure of the angles at these vertices. However, we will often use a Greek letter, such as α (alpha), β (beta) or θ (theta) to do so. The corresponding lower-case letters, a, b and c, represent the lengths of the sides opposite the vertices. For example, b represents the length of the side opposite angle B, that is, the line segment AC, or $[AC]$ (Figure 8.1).

In a right triangle, the longest side is opposite the right angle (i.e. measure of 90°) and is called the **hypotenuse**, and the two shorter sides adjacent to the right angle are often called the **legs** (Figure 8.2). Because the sum of the three angles in any triangle in plane geometry is 180°, then the two non-right angles are both **acute angles** (i.e. measure between 0 and 90 degrees). It also follows that the two acute angles in a right triangle are a pair of **complementary angles** (i.e. have a sum of 90°).

Figure 8.2 Right triangle terminology.

Trigonometric functions of an acute angle

We can use properties of similar triangles and the definitions of the sine, cosine and tangent functions from Chapter 7 to define these functions in terms of the sides of a right triangle.

Figure 8.3 Trigonometric functions defined in terms of sides of similar triangles.

The right triangles shown in Figure 8.3 are **similar triangles** because corresponding angles have equal measure – each has a right angle and an acute angle of measure θ. It follows that the ratios of corresponding sides are equal, allowing us to write the following three proportions involving the sine, cosine and tangent of the acute angle θ.

$$\frac{\sin\theta}{1} = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \frac{\cos\theta}{1} = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \frac{\tan\theta}{1} = \frac{\sin\theta}{\cos\theta} = \frac{\text{opposite}}{\text{adjacent}}$$

The definitions of the trigonometric functions in terms of the sides of a right triangle follow directly from these three equations.

Right triangle definition of the trigonometric functions

Let θ be an **acute angle** of a right triangle, then the sine, cosine and tangent functions of the angle θ are defined as the following ratios in the right triangle:

$$\sin\theta = \frac{\text{side opposite angle }\theta}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{side adjacent angle }\theta}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{side opposite angle }\theta}{\text{side adjacent angle }\theta}$$

It follows that the sine, cosine and tangent of an acute angle are positive.

It is important to understand that properties of similar triangles are the foundation of right triangle trigonometry. Regardless of the size (i.e. lengths of sides) of a right triangle, so long as the angles do not change, the ratio of any two sides in the right triangle will remain *constant*. All the right triangles in Figure 8.4 have an acute angle with a measure of 30° (thus, the other acute angle is 60°). For each triangle, the ratio of the side opposite the 30° angle to the hypotenuse is exactly $\frac{1}{2}$. In other words, the sine of 30° is always $\frac{1}{2}$. This agrees with results from the previous chapter, knowing that an angle of 30° is equivalent to $\frac{\pi}{6}$ in radian measure.

Thales of Miletus (circa 624–547) was the first of the Seven Sages, or wise men of ancient Greece, and is considered by many to be the first Greek scientist, mathematician and philosopher. Thales visited Egypt and brought back knowledge of astronomy and geometry. According to several accounts, Thales, with no special instruments, determined the height of Egyptian pyramids. He applied formal geometric reasoning. Diogenes Laertius, a 3rd-century biographer of ancient Greek philosophers, wrote: 'Hieronymus says that [Thales] even succeeded in measuring the pyramids by observation of the length of their shadow at the moment when our shadows are equal to our own height.' Thales used the geometric principle that the ratios of corresponding sides of similar triangles are equal.

27 The Eiffel Tower in Paris is 300 metres high (not including the antenna on top). What will be the angle of elevation of the top of the tower from a point on the ground (assumed level) that is 125 metres from the centre of the tower's base?

28 A 1.62-metre tall woman standing 3 metres from a streetlight casts a 2-metre long shadow. What is the height of the streetlight?

29 A pilot measures the angles of depression to two ships to be 40° and 52° (see the figure). If the pilot is flying at an elevation of 10 000 metres, find the distance between the two ships.

30 Find the measure of all the angles in a triangle with sides of length 8 cm, 8 cm and 6 cm.

31 From a 50-metre observation tower on the shoreline, a boat is sighted at an angle of depression of 4° moving directly toward the shore at a constant speed. Five minutes later the angle of depression of the boat is 12°. What is the speed of the boat in kilometres per hour?

e diagram.
ificant figures.

cted from an anchor point on level ground to
ire makes a 65° angle with the ground at the
farther from the tower than the wire's anchor
ower, the angle of elevation to the top of the
the nearest tenth of a metre.

p of a hill. The angles of elevation of the
m the same spot at the base of the hill are
tively. Relative to its base, how high is the hill

a vertical pole as seen from a point
uble its angle of elevation as seen from a
the height (to the nearest tenth of a metre)
bserver's eyes.

Hint boxes can be found alongside questions, exercises, worked examples and new topics; they identify common pitfalls and provide insights into how students can achieve the highest marks in an examination.

These boxes contain interesting information and are designed to increase students' wider knowledge. Also found throughout the book are green Key term boxes to highlight key facts from the main text, and pink Theory of Knowledge boxes to stimulate thought and discussion in context. There is also an innovative Theory of Knowledge chapter at the end of the book.

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